APPM 2350 - Solar Panels and Optimization

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1 The Weather on Suluclac

The weather on the island of Suluclac can be described by the following cloudiness function:

$$C(s,t) = \frac{3 - [1 + (s-0.2)^2]\cos^2 t}{3} \tag{1}$$

where s is the angle between the sun rays and the plane of the equator at noon and t is the angle proportional to the time of day. The angle s corresponds with the season of year, with the winter solstice occurring at s= -23°= -23π/180 radians or about 0.4 radians, the summer solstice occurring at s= 23°, and the autumnal and vernal equinoxes occurring at s = 0. The angle t corresponds with the time of day, such that t= $-\pi/2$ radians at dawn, t= 0 radians at noon, and t = $\pi/2$ radians at dusk.

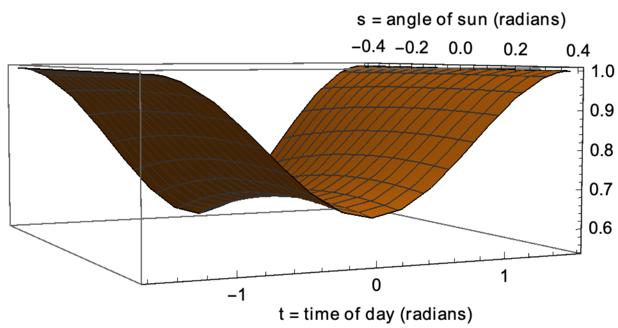

Figure 1: A 3D plot of C(s,t) over the domain D:[$-\pi/2 \leq t \leq \pi/2, -0.4 \leq s \leq$ 0.4]

As shown in the 3D plot of C(s,t) above, the cloudiness on Suluclac generally decreases from dawn until noon and increases from noon until dusk. Over the course of the year, the island is most cloudy when s is around -0.4 radians, which corresponds to winter, and the least cloudy when s is around 0.4 radians which corresponds to summer. The cloudiness generally increases as seasons change from summer to winter, and decreases as the seasons change from winter to summer.

2 The Critical Points

2.1 Visualizing the energy collected by the solar panel each day

The energy collected by the solar panel each day (kilowatt-hours per square meter per day, kW-h/m^2/day) is given by:

$$W(s,u) = 1 + (1 + 0.65s - 1.2s^2 - 0.4s^3 + 0.35s^4)\cos u \\ + (1.4s - 0.4s^2 - 1.5s^3 - 0.35s^4)\sin u \quad (2)$$

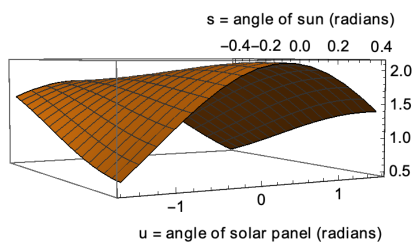

Figure 2: A 3D plot of W(s,u) over the domain D $[-\pi/2 \leq u \leq \pi/2, -0.4 \leq s \leq 0.4]$

2.2 Determining an initial guess for the critical point

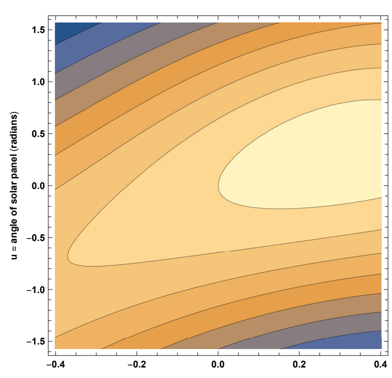

Figure 3: A contour plot of W(s,u) over the domain D $[-\pi/2 \leq u \leq \pi/2, -0.4 \leq s \leq 0.4]$

From the contour plot, we determined our initial guess for a possible critical point to be $(0.3, 0.3)$ because this lies at the heart of the highest topographical region so we think the global maximum is around the point $(0.3, 0.3)$.

2.3 Determining and classifying the critical point

Using the FindRoot function in Mathematica, we determined the critical point to occur at $(s, u) = (0.32402, 0.320456)$. We then classified the critical point by evaluating the second derivative at the critical point.

$$W_{ss}(s, u) \;=\; (-2.4 - 2.4s + 4.2s^2)Cos[u] + (-0.8 - 9.s - 4.2s^2)Sin[u] \quad (3)$$

Because $W_{ss}(0.32402, 0.320456) =$ (-3.90687* -1.13253) - (0.654^2) ≤ 0. Since the $W_{ss} < 0$, the critical point is at a local maximum determined by using the second derivative test.

2.4 Determining the energy at the critical point

From evaluating W(s,u) at $(0.32402, 0.320456)$, we determined the energy collected by the solar panel to be 2.13253 kWh/m^2/day at this point.

3 The Extreme Points

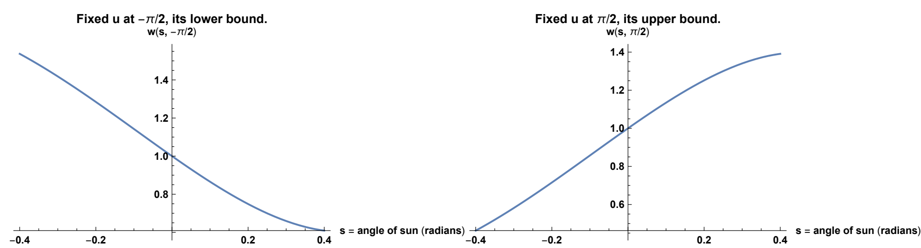

Figure 4: Side views of u= $-\pi/2$ and u=$\pi/2$ edges

It appears obvious there are no critical points on the u= $-\pi/2$ or u= $\pi/2$ edges of W(s,u). This can be seen from a side view of our 3D plot (Figure 2) and from the plots above. Because the functions on these edges do not change from increasing to decreasing or from decreasing to increasing, there is no horizontal tangent line and therefore no critical points. This is shown to be true when solving the derivative for 0 only results outside of bounds can be found therefor we can conclude there are no critical points in the regions when u = $-\pi/2$ or u = $\pi/2$.

We then proceeded to evaluate W(s,u) on the other two boundaries. For a fixed s of -0.4 radians, we determined the critical point to occurs at u = -0.744689. Based on the graph, W(-0.4, -0.744689) = 1.79228 is a maximum on the s = -0.4 boundary. For a fixed s of 0.4 radians, we determined the critical point to occur at u = 0.356083. Based on the graph, W(0.4, 0.356083) = 2.12173 is a maximum on the s = 0.4 boundary.

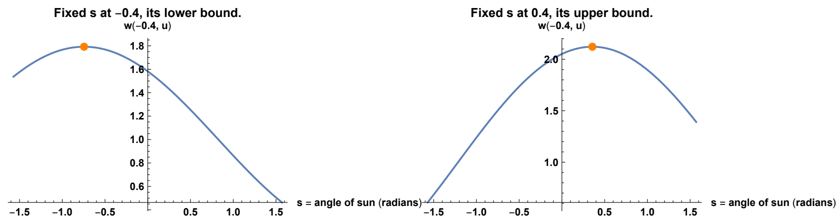

Figure 5: Side views of s=-0.4 and s=0.4 edges

4 Energy at Critical Points

Corner Points	W(s,u)	Critical Points	W(s,u)
(-0.4, $-\pi/2$)	1.53696	(0.32402, 0.320456)	2.13253
(0.4, $-\pi/2$)	0.60896	(-0.4, -0.744689)	1.79228
(-0.4, $\pi/2$)	0.46304	(0.4, -0.356083)	2.12173
(0.4, $\pi/2$)	1.39104		

After evaluating W(s,u) at our corner points and the critical points on our boundary, we determined our global maximum to be 2.13253 and our global minimum to be 0.46304. For the given domain, D = $[-0.4 \leq s \leq 0.4, -\pi/2 \leq u \leq \pi/2]$, a global minimum of 0.46304 occurs at the corner s = -0.4 and u = $\pi/2$. For the given domain, D = $[-0.4 \leq s \leq 0.4, -\pi/2 \leq u \leq \pi/2]$, a global maximum of 2.13253 occurs at the critical point s= 0.32402 and u = 0.320456

5 Which season will we end up collecting the most solar energy?

From March 21-June 21, 0 < s ≤ 0.4 radians and from June 22-September 22, 0.4 > s ≥ 0. As shown in the contour plot of W(s,u) (Figure 3), the solar panel collects the most energy when s is between 0.2 and 0.4 radians. More specifically, the solar panel will collect the most energy in the months surrounding the time when s = 0.3 radians, which corresponds to late-spring and early-summer.

6 Determining the Optimal Angle for u

Our optimal path makes sense based on our 3D graph of W(s,u). For a given point, the direction of the gradient of W(s,u) gives the direction of fastest increase of f. This creates a path which is perpendicular to the level curves of the contour plot. The optimal angle for u should be along this curve of greatest increase in order to maximize the energy collected by the solar panel.

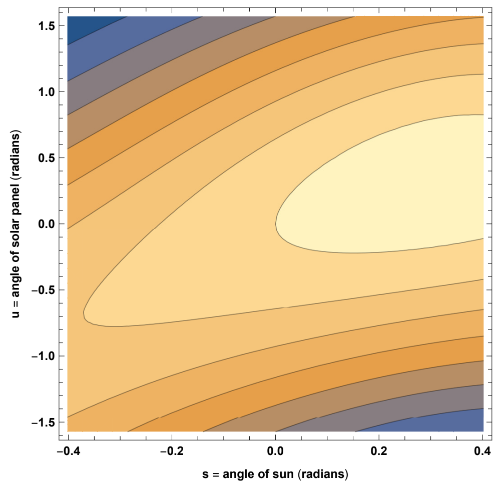

Figure 6: A contour plot of W(s,u) over the domain D $[-\pi/2 \leq u \leq \pi/2, -0.4 \leq s \leq 0.4]$

7 What Time of Year Should we Replace the Solar Panels?

The contour plot of W(s,u) (Figure 3) is lowest at s= -0.4 radians. This value of s corresponds with the winter solstice, which occurs on December 21. Because the solar panel collects the least energy at this time, the best time to replace solar panels to minimize energy loss is around the winter solstice.

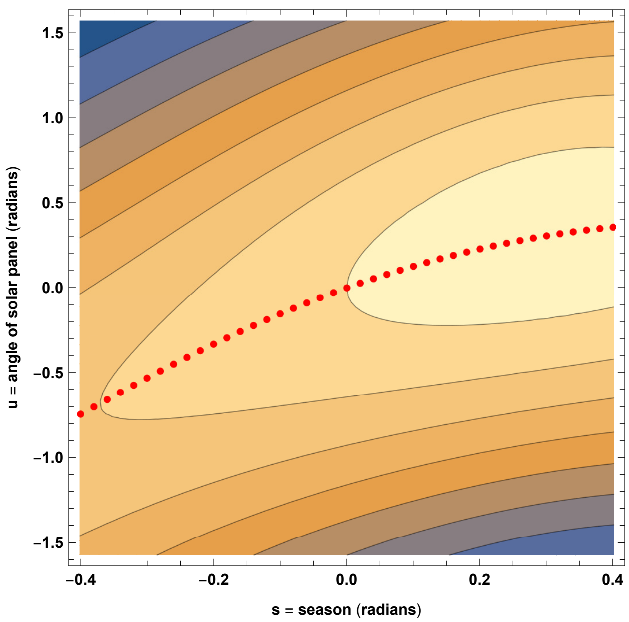

Figure 7: A contour plot of W(s,u) over the domain D $[-\pi/2 \leq u \leq \pi/2, -0.4 \leq s \leq 0.4]$, with the red dots representing the optimal angle for u that will maximize the amount of solar energy collected by the solar panel.